2017 Spring PHYS2320 電磁學（Electromagnetism） Final 滿分 100

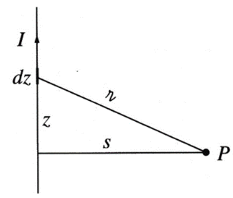
[Griffiths Chs. 10 and 12] 2018/06/12, 10:10am – 12:00am, 教師：張存續

1. Write the equations (if possible) and explain the following terms as clear as possible.
   (a) Lorentz gauge and Coulomb gauge. (4%)
   (b) Gauge transformations and gauge freedom. (4%)
   (c) Lienard-Wiechert potentials. (4%)
   (d) The two postulates of the special relativity (4%)
   (e) Conserved quantity and invariant quantity. (4%)

2. (a) The transformations between two inertial systems S and $\bar{S}$ are $\bar{x} = \gamma(x - vt)$ and $\bar{t} = \gamma(t - vx/c^2)$. Show that when $\Delta t = 0$, $\Delta x = \Delta\bar{x}/\gamma$; but when $\Delta\bar{t} = 0$, $\Delta\bar{x} = \Delta x/\gamma$. Explain why the length relations depend on the simultaneity. (10%)
   (b) Show that $(E^2 - c^2B^2)$ is relativistically invariant. (10%)

3. Show that the retarded potential satisfy the Lorentz gauge condition. (20%)
   [Hint: the retarded potentials $V(\mathbf{r},t) = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int \dfrac{\rho(\mathbf{r}',t_r)}{\mathscr{r}}d\tau'$ and $\mathbf{A}(\mathbf{r},t) = \dfrac{\mu_0}{4\pi}\displaystyle\int \dfrac{\mathbf{J}(\mathbf{r}',t_r)}{\mathscr{r}}d\tau'$.]

4. (20%) An infinite straight wire carriers a linearly increasing current: $I(t) = kt$ for $t > 0$.
(a) Find the scalar and vector potentials. (10%)
(b) Find the electric field generated. (10%)



5. The Maxwell equations can be written in terms of the field tensor.

$$F^{\mu\nu} = \begin{Bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{Bmatrix} \qquad \text{[Hint: } t^{\mu\nu} = \begin{Bmatrix} t^{00} & t^{01} & t^{02} & t^{03} \\ t^{10} & t^{11} & t^{12} & t^{13} \\ t^{20} & t^{21} & t^{22} & t^{23} \\ t^{30} & t^{31} & t^{32} & t^{33} \end{Bmatrix}\text{].}$$

(a) Find the corresponding Maxwell's equation for $\dfrac{\partial F_{\mu\nu}}{\partial x^{\lambda}} + \dfrac{\partial F_{\lambda\mu}}{\partial x^{\nu}} + \dfrac{\partial F_{\nu\lambda}}{\partial x^{\mu}} = 0$, when $(\lambda,\mu,\nu) = (1,2,3)$. (10%)
(b) Find the corresponding Maxwell's equation for $\dfrac{\partial F^{\mu\nu}}{\partial x^{\nu}} = \mu_0 J^{\mu}$, when μ=1, 2, and 3. (10%)